2024 MMU Spring Competition (3-5)

You will have 30 minutes to complete as much of this test as you can. There are 20 free response questions total, and questions are arranged roughly from easiest to most difficult. Units are not needed. Write answers on the given line below each question. Calculators are not allowed. Do not begin the test until told to do so. Good Luck!

Full Name: _____

Grade: _____

Age: _____

1. _____ What is the largest number with at most one repeated digit whose digits add up to 14?

2. _____pairs_____ The number 174 can be written as the product of three primes. What is the sum of the three primes? A prime number is a number whose only divisors are 1 and itself.

3. _____ What is the value of a such that the quadratic $x^2 + ax + 18$ has 1 real root? Express your answer in simplest radical form

4. _____words_____ How many words can you make by rearranging the letters in MISSION? For example, MISSION, SSIIONM, and MISISON are three different words that can be made.

5. _____songs_____ Mr. East loves producing music. He has two playlists of twenty equally long songs each, lasting for 60 minutes and 120 minutes, respectively. If he shuffles both playlists into a new playlist and plays the new playlist for exactly 54 minutes, what is the expected number of songs that will be played in that time?

6. _____ If A and B are real numbers such that $A + B = A \times B = x$, and $x > 1$, what is $\sqrt{A^2 + B^2 + 1}$ in terms of x?

7. _____ Mingchuan is making brownies. He has 576 chocolate chips. He bakes a 5 by 6 grid of cookies in a rectangular pan, but he puts twice as many chocolate chips into the edge cookies as the interior cookies (edge cookies are cookies that are touching the edge of the pan, including corner cookies) . How many chocolate chips does each of the interior cookies have?

8. _____ What is the greatest common divisor of 3017 and 3094?

9. _____ A monochromatic rectangle is defined as a rectangle whose four corners are dots of the array and are all of the same color. Given that we are coloring all the dots on a $5 \times n$ array of dots either white or black, what is the minimum value of n such that there is always guaranteed to have a monochromatic rectangle, no matter how the array is colored?

10. _____ A pizza delivery worker mixed up where he is supposed to deliver his pizzas. He has 5 houses and 5 pizzas, and each house ordered exactly 1 pizza. What is the chance he delivers none of the pizzas correctly?

11. _____ Albert, Bryan, Caleb, and David are rolling a fair 4 sided die in that order. A player wins when they roll a number corresponding to their place in that order (1 for Albert, 2 for Bryan, 3 for Caleb, 4 for David). What is the chance that David wins if the game continues until someone wins?

12. _____ If the game of Tfarcenim allows a player to score $4, 7, 15, 19, 23, 24$, or 36 points, and all players start at 0 points, what is the largest integer score that cannot be made?

13. _____ Let a equal the infinite geometric sequence $3^1 + 3^0 + 3^{-1} + 3^{-2} + ...$ Let b equal the infinite geometric sequence $4^0 + 4^{-1} + 4^{-2} + 4^{-3} + ...$ What is $a - b$? Express your answer as a common fraction.

14. _____ A regular octagon is inscribed in a circle. If four of the eight vertices are randomly chosen, what is the probability that the shape made with the four vertices is a rectangle? Express your answer as a common fraction.

15. _____ What is the largest prime divisor of $12 \times 11 \times 10 \times 9 - 8 \times 7 \times 6 \times 5 + \times 4 \times 3 \times 2 \times 1$?

16. _____ Suppose you have the set of six numbers: 1,2,3,4,5,6. In how many ways can you arrange these six numbers so that 4 appears before 5, which appears before 6? For example, 1,4,2,5,3,6 is a valid case, but 2,6,3,1,4,5 is not a valid case.

17. _____ A player flips 6 coins and rolls a typical 6 sided die. What is the chance that the number of heads is equal to the number rolled on the die?

18. _____ How many ordered sets of distinct positive integers a, b, c are there such that the product of those three numbers is equal to the product of $9, 14, 19$, and 29?

19. _____ Let a, b, c, d, e be positive integers such that

$$a + \frac{1}{b + \frac{1}{c + \frac{1}{d + \frac{1}{e}}}} = \frac{972}{471}.$$

Find $a \times b \times c \times d \times e$.

20. _____ units^2 Equiangular octagon $PENTAGON$ has $PE = 3, EN = 4, NT = 2, TA = 6, AG = 3, GO = 4, ON = 3$ and $NP = 3$. Find the area of $PENTAGON$.