

2024 MMU Spring Competition (6-8)

You will have 40 minutes to complete as much of this test as you can. There are 30 free response questions total, and questions are arranged roughly from easiest to most difficult. Units are not needed. Write answers on the given line below each question. Calculators are not allowed. Do not begin the test until told to do so. Good Luck!

Full Name: _____

Grade: _____

Age: _____

1. _____ What is the largest number with at most one repeated digit whose digits add up to 14?
2. _____ What is the value of a such that the quadratic $x^2 + ax + 18$ has 1 real root? Express your answer in simplest radical form
3. _____ words How many words can you make by rearranging the letters in MISSION? For example, MISSION, SSIIONM, and MISISON are three different words that can be made.
4. _____ pounds If passing a test is defined by getting at least $3/5$ of the questions correct, and David is equally likely to answer a question correctly or incorrectly, what is the probability that David got all 5 questions correct given that he passed the test? Express your answer as a common fraction.
5. _____ choc chips Mingchuan is making brownies. He has 576 chocolate chips. He bakes a 5 by 6 grid of cookies in a rectangular pan, but he puts twice as many chocolate chips into the edge cookies as the interior cookies (edge cookies are cookies that are touching the edge of the pan, including corner cookies) . How many chocolate chips does each of the interior cookies have?
6. _____ units Let a equal the infinite geometric sequence $3^1 + 3^0 + 3^{-1} + 3^{-2} + \dots$. Let b equal the infinite geometric sequence $4^0 + 4^{-1} + 4^{-2} + 4^{-3} + \dots$. What is $a - b$? Express your answer as a common fraction.

7. _____ What is the greatest common divisor of 3017 and 3094?
8. _____ books Ashlee has some books on her shelf. Ashley has 4 books. Ashleigh has the average number of books Ashlee and Ashley have, and Ash-Li has the average number of books Ashley and Ashleigh have. If Ash-Li has 6 books, how many books does Ashlee have?
9. _____ A monochromatic rectangle is defined as a rectangle whose four corners are dots of the array and are all of the same color. Given that we are coloring all the dots on a $5 \times n$ array of dots either white or black, what is the minimum value of n such that there is always guaranteed to have a monochromatic rectangle, no matter how the array is colored?
10. _____ Albert, Bryan, Caleb, and David are rolling a fair 4 sided die in that order. A player wins when they roll a number corresponding to their place in that order (1 for Albert, 2 for Bryan, 3 for Caleb, 4 for David). What is the chance that David wins if the game continues until someone wins?
11. _____ songs Mr. East loves producing music. He has two playlists of twenty equally long songs each, lasting for 60 minutes and 120 minutes, respectively. If he shuffles both playlists into a new playlist and plays the new playlist for exactly 54 minutes, what is the expected number of songs that will be played in that time?
12. _____ A regular octagon is inscribed in a circle. If four of the eight vertices are randomly chosen, what is the probability that the shape made with the four vertices is a rectangle? Express your answer as a common fraction.

13. _____ The parabola defined by $x^2 + ax + b$ has the property that when you shift it to the right by 4 units and down by 12 units, you get the parabola $bx^2 + ax + B$. Find $|a| + |b|$
14. _____ Find the largest prime factor of $1 + 2^{13} + 2^{24}$.
15. _____ Suppose you have the set of six numbers: 1,2,3,4,5,6. In how many ways can you arrange these six numbers so that 4 appears before 5, which appears before 6? For example, 1,4,2,5,3,6 is a valid case, but 2,6,3,1,4,5 is not a valid case.
16. _____ A player flips 6 coins and rolls a typical 6 sided die. What is the chance that the number of heads is equal to the number rolled on the die? Express your answer as a common fraction.
17. _____ sets How many ordered sets of distinct positive integers a, b, c are there such that the product of those three numbers is equal to the product of 9, 14, 19, and 29?
18. _____ units² Circles $\omega_1, \omega_2, \omega_3$ are mutually tangent to each other, and have radius of $\frac{1}{18}, \frac{1}{23},$ and $\frac{1}{27}$ in that order. Let r_4, r_5 denote the radii of circles ω_4, ω_5 . Given that ω_4 is the circle that is internally tangent to all three circles, and ω_5 is the circle that is externally tangent to all three circles, find $\frac{1}{r_4} \times \frac{1}{r_5}$.

19. _____ Polynomial $f(x)$ is defined by

$$f(x) = x^{10} - 4x^9 + 8x^8 - 13x^7 + 15x^6 - 17x^5 + 15x^4 - 13x^3 + 8x^2 - 4x + 1.$$

Find the sum of all real roots of $f(x)$. Express your answer as a common fraction in simplest radical form.

20. _____ Let $S = \{1, 2, 3, \dots, 2024\}$. The total number of subsets T for which the sum of all elements in T is divisible by 23 can be expressed in the form

$$\frac{a^b + cd^e}{f},$$

where a, b, c, d, e, f are not necessarily distinct positive integers. Find $a+b+c+d+e+f$.